ProxQP: Yet another Quadratic Programming Solver for Robotics and beyond

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a joint work with Sarah El-Kazdadi, Adrien Taylor and Justin Carpentier

Introduction

Convex Quadratic Program (QP)

$$\min_{x \in \mathbb{R}^n} \underbrace{\frac{1}{2} x^\top H x + q^\top x}_{:= f(x)}$$

$$\text{s.t. } Cx - u \leqslant 0.$$

$$H \in \mathcal{S}_+(\mathbb{R}^n), C \in \mathbb{R}^{n_i \times n}$$

Convex QP is a core modelling tool of robotics:

- Friction-less unilateral **contact modelling**
- Constrained **forward dynamics**
- **Inverse kinematics and dynamics**
- **Legged locomotion**
- Constrained **optimal control**
- …

Introduction

Current QP solvers have trouble being on the same time accurate, fast and numerically robust.

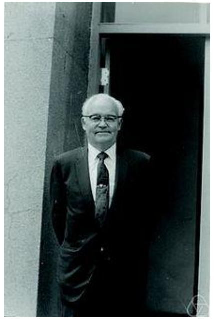
QP solvers	Method used	Limitations
Mosek, Gurobi	Interior Point	No warm-start
OSQP	ADMM	Accuracy to low threshold
quadprog, qpOASES	Active set	Robustness

Augmented Lagrangian Methods

The Augmented Lagrangian

$$\mathcal{L}_A(x, z; \mu) \stackrel{\text{def}}{=} f(x)$$
$$+ \frac{1}{2\mu}\left(\|[Cx - u + \mu z]_+\|_2^2 - \|\mu z\|_2^2\right)$$

Smoothed and shifted penalization



Magnus Hestenes

Michael J.D. Powell

Augmented Lagrangian Methods

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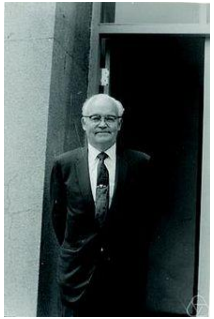
Method of Multipliers

$$\begin{cases} x^{k+1} \approx_{\varepsilon^k} & \text{argmin}_x \, \mathcal{L}_A(x, z^k; \mu), \\ z^{k+1} = & [z^k + \frac{1}{\mu}(Cx^{k+1} - u)]_+. \end{cases}$$



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The Proximal Augmented Lagrangian

$$\Phi^k_{\mu,\rho}(x) \stackrel{\text{def}}{=} \mathcal{L}_A(x, z^k; \mu) + \frac{\rho}{2}\|x - x^k\|_2^2$$

Proximal term

Method of Multipliers

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Proximal Method of Multipliers

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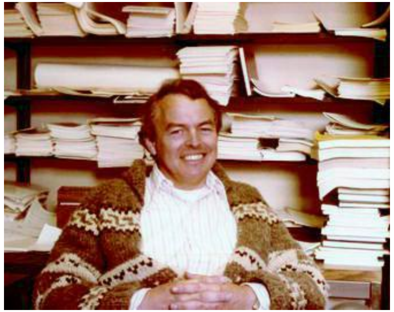

R. Tyrrell Rockafellar

Augmented Lagrangian Methods

The Augmented Lagrangian

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The Proximal Augmented Lagrangian

$$\Phi_{\mu,\rho}^k(x) \stackrel{\text{def}}{=} \mathcal{L}_A(x, z^k; \mu) + \frac{\rho}{2}\|x - x^k\|_2^2$$

First contribution:
the Proximal Primal Dual Augmented Lagrangian

$$\mathcal{M}_{\mu,\rho}^k(x, z) := \Phi_{\mu,\rho}^k(x) + \frac{1}{2\mu}\|[Cx - u + z^k\mu]_+ - z\mu\|_2^2.$$

Method of Multipliers

$$\begin{cases} x^{k+1} \approx_{\varepsilon^k} & \operatorname{argmin}_x \mathcal{L}_A(x, z^k; \mu), \\ z^{k+1} = & [z^k + \frac{1}{\mu}(Cx^{k+1} - u)]_+. \end{cases}$$

Proximal Method of Multipliers

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More robust and accurate

Augmented Lagrangian Methods

The Augmented Lagrangian

$$\mathcal{L}_A(x, z; \mu) \overset{\text{def}}{=} f(x) + \frac{1}{2\mu}\left(\|[Cx - u + \mu z]_+\|_2^2 - \|\mu z\|_2^2\right)$$

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First step of ProxQP

$$x^{k+1}, z^{k+1} \approx_{\varepsilon^k} \underset{x,z}{\text{argmin}}\, \mathcal{M}_{\mu,\rho}^k(x, z).$$

ProxQP algorithm

while *Stopping criterion not satisfied* **do**

$\hat{x}, \hat{z} \approx_{\varepsilon^k} \operatorname{argmin}_{x,z} \mathcal{M}^k_{\mu,\rho}(x, z)$;

$x^{k+1} = \hat{x}$;

if $\|[Cx^{k+1} - u]_+\|_\infty \leqslant \eta_k$ **then**

Accept multiplier $\hat{z}$;

Strictly decrease ε^k, η^k;

else

Keep previous multiplier z^k;

Low decrease of ε^k, η^k;

Strictly decrease μ;

end

end

Second contribution: **mixing our new merit function with BCL**

Bound Constrained Augmented Lagrangian (BCL) **= globalization strategy**

ProxQP solver implementation

- **Fast: C++ implementation**, using **Eigen** for efficient linear algebra
- **Scalable: unique solver with 3 specialized backends** (**dense**, **sparse** and **very large matrices)**
- **Easy to use: API close to OSQP**, with **python bindings**
- **Open-source: BSD license**

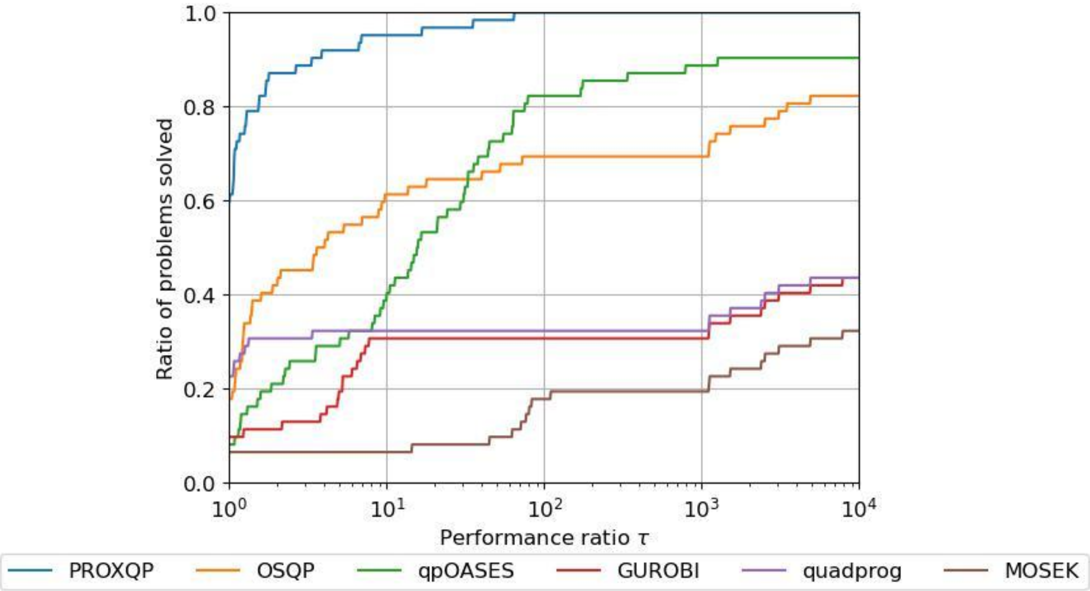
https://github.com/Simple-Robotics/proxsuite

Benchmarks

	OSQP	PROXQP
Inverse kinematics (μs)	167 ± 93	24 ± 7
Inverse dynamics (μs)	441 ± 193	25 ± 6

Solving times for controlling the center of mass of TALOS using Pinocchio library

Benchmarks

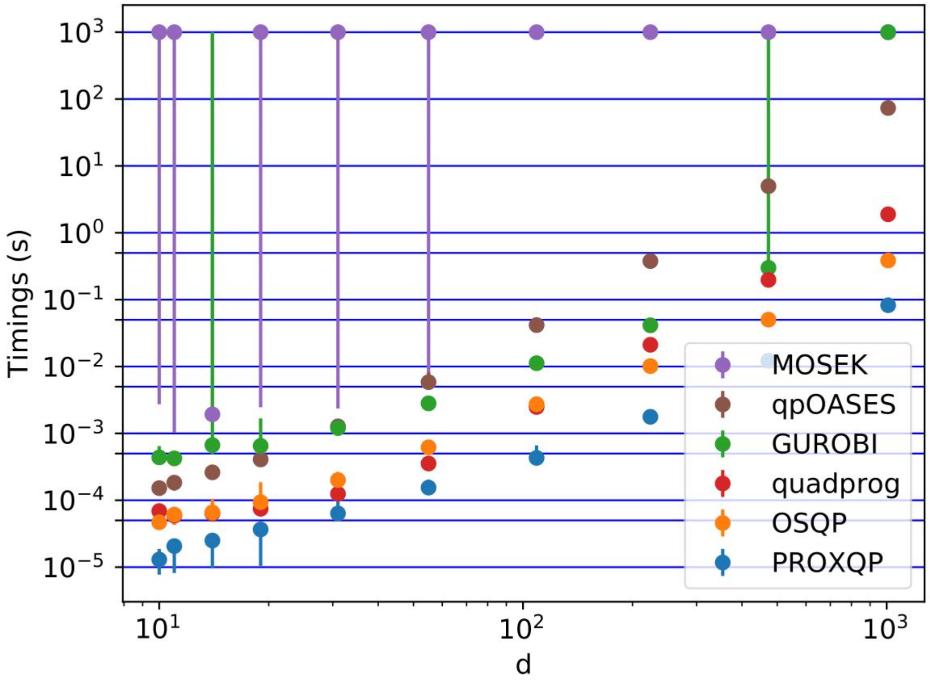


$\text{sparsity} \leqslant 10\%$

$\text{matrix dimensions} \leqslant 1000$

Performance profiles for **very hard QPs** from the optimization community (**the higher the better**)

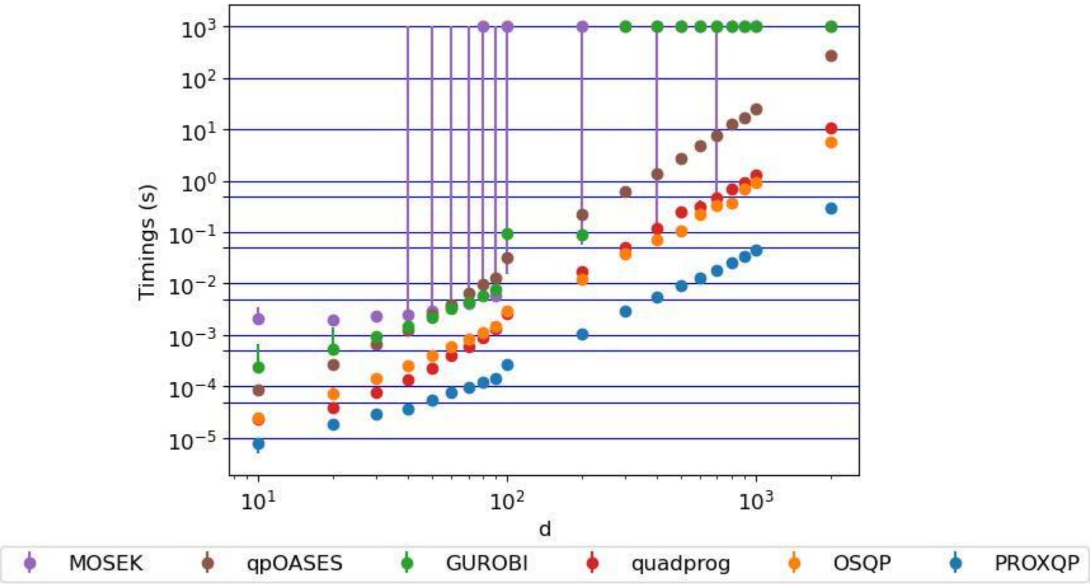
Benchmarks



$$\text{sparsity} = \frac{\text{number of non zeros}}{\text{number of matrix elements}}$$

Solving times for equality and inequality constrained QPs with 15% of sparsity

Benchmarks



Dense problems

matrix dimensions $\leqslant 1000$

Solving times for **standard QPs** from the optimization community (**the lower the better**)

- **New Augmented Lagrangian Algorithm**
  - new merit function
  - globalization strategy
- **ProxQP = fast, robust, accurate and open source solver**

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https://github.com/Simple-Robotics/proxsuite

Inria

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